



预测科学研究中心
The Centre for Forecasting Science



Data
Lab for
Social Good
Cardiff University, UK

Conformal Inference for Time Series Forecasting and Its Implementations in R

Xiaoqian Wang¹ & Jesus Diaz Martinez²

1. The Center for Forecasting Science, Chinese Academy of Sciences (CAS)
2. King Abdullah University of Science and Technology (KAUST)

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- 1 Brief Overview of Conformal Prediction
- 2 Conformal Prediction for Multi-step Forecasting
- 3 Theoretical Properties
- 4 Empirical Evaluation
- 5 Takeaways & Future Work
- 6 Hands-on Exercises in R (by Jesus)

Model-based Approaches

- Examples: ARIMA/state-space models
- Assumption: Explicit parametric error distribution
- Limitations: Sensitive to misspecification, coverage invalid under non-stationarity

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Resampling & Bayesian Approaches

- Bootstrap: Residual bootstrap, block bootstrap (autocorrelation)
- Bayesian: Posterior predictive distribution
- Limitations: Computationally heavy, depend on prior / resampling scheme, no finite-sample guarantees

Model-dependent / Heuristic Approaches

- Quantile methods: Quantile regression, ML-based quantile models
- Heuristic ML: Ensembles, MC dropout
- Limitations: Model misspecification impacts interval accuracy, no finite-sample guarantees, calibration challenging for time series

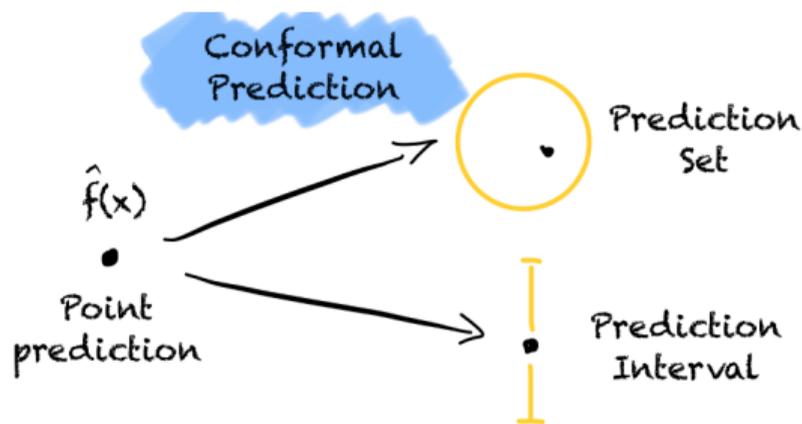
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Conformal Prediction

- Distribution-free
- Model-agnostic
- Finite-sample coverage

Conformal Prediction (a.k.a. Conformal Inference)



Conformal prediction (Vovk et al., 2005) is an algorithm for uncertainty quantification that produces statistically valid prediction regions for any underlying point predictor only assuming exchangeability of the data.

(from Wikipedia)

Classical Conformal Prediction Methods

In regression settings:

- The **data** $Z_i = (X_i, Y_i)$ are assumed to be exchangeable.
- The **algorithm** \mathcal{A} which maps data to a fitted model $\hat{\mu} : \mathcal{X} \rightarrow \mathbb{R}$ is assumed to treat the data points symmetrically.
- The **aim** is to construct a prediction set $\hat{\mathcal{C}}_{n+1}(X_{n+1})$ satisfying
$$\mathbb{P} \left\{ Y_{n+1} \in \hat{\mathcal{C}}_n(X_{n+1}) \right\} \geq 1 - \alpha.$$

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1 **Split / Inductive Conformal Prediction** (Vovk et al., 2005)

2 Full / Transductive Conformal Prediction (Vovk et al., 2005)

3 Jackknife+ (Barber et al., 2021)

Split / Inductive Conformal Prediction

- 1 training data set: pre-trained model $\hat{\mu} : \mathcal{X} \rightarrow \mathbb{R}$.
- 2 calibration / holdout set: nonconformity scores $R_i = |Y_i - \hat{\mu}(X_i)|$, $i = 1, \dots, n$.
- 3 prediction set for a given level α :

$$\hat{C}_n(X_{n+1}) = \hat{\mu}(X_{n+1}) \pm Q_{1-\alpha} \left(\sum_{i=1}^n \frac{1}{n+1} \cdot \delta_{R_i} + \frac{1}{n+1} \cdot \delta_{+\infty} \right).$$

Drawback: The loss of accuracy due to sample splitting, sensitive to calibration set, the length of intervals is fixed.

Full / Transductive Conformal Prediction

1 training data & a hypothesized test point:

$$\hat{\mu}^y = \mathcal{A}((X_1, Y_1), \dots, (X_n, Y_n), (X_{n+1}, y)) \text{ for each } y \in \mathbb{R}.$$

2 residuals: $R_i^y = \begin{cases} |Y_i - \hat{\mu}^y(X_i)|, & i = 1, \dots, n \\ |y - \hat{\mu}^y(X_{n+1})|, & i = n + 1 \end{cases}$.

3 prediction set: $\hat{C}_n(X_{n+1}) = \left\{ y \in \mathbb{R} : R_{n+1}^y \leq Q_{1-\alpha} \left(\sum_{i=1}^{n+1} \frac{1}{n+1} \cdot \delta_{R_i^y} \right) \right\}$.

Drawback: a steep computational cost.

THEOREM:

- $\mathbb{P} \left\{ Y_{n+1} \in \hat{C}_n(X_{n+1}) \right\} \geq 1 - \alpha$ holds true for both split conformal and full conformal.

Jackknife+

1 training data with i th point removed:

$$\hat{\mu}_{-i} = \mathcal{A}((X_1, Y_1), \dots, (X_{i-1}, Y_{i-1}), (X_{i+1}, Y_{i+1}), \dots, (X_n, Y_n)), i = 1, \dots, n.$$

2 residuals: $R_i^{\text{LOO}} = |Y_i - \hat{\mu}_{-i}(X_i)|, i = 1, \dots, n.$

3 prediction set:

$$\left[Q_\alpha \left(\sum_{i=1}^n \frac{1}{n+1} \cdot \delta_{\hat{\mu}_{-i}(X_{n+1}) - R_i^{\text{LOO}}} + \frac{1}{n+1} \cdot \delta_{-\infty} \right), Q_{1-\alpha} \left(\sum_{i=1}^n \frac{1}{n+1} \cdot \delta_{\hat{\mu}_{-i}(X_{n+1}) + R_i^{\text{LOO}}} + \frac{1}{n+1} \cdot \delta_{+\infty} \right) \right]$$

THEOREM:

■ $\mathbb{P} \left\{ Y_{n+1} \in \hat{C}_n(X_{n+1}) \right\} \geq 1 - 2\alpha$ holds true for jackknife+.

Conformal Prediction Beyond Exchangeability

- Covariate shift (Lei & Candès, 2021; Tibshirani et al., 2019; Yang et al., 2024)
- Distribution drift (Gibbs & Candès, 2021; Zaffran et al., 2022)
- Spatial dependence (Mao et al., 2024)
- **Temporal dependence**
 - ▶ Ensemble batch prediction intervals (EnbPI, Xu & Xie, 2021)
 - ▶ Weighted / Locally exchangeable conformal prediction (Barber et al., 2023)
 - ▶ Adaptive conformal prediction and its extensions (Bastani et al., 2022; Gibbs & Candès, 2021; Gibbs & Candès, 2024; Zaffran et al., 2022)
 - ▶ Quantile tracking (Angelopoulos et al., 2023)

Limitations of Conformal Prediction for Time Series

1 Assumption of (local) exchangeability

- Weighted / locally exchangeable CP relaxes exchangeability, but:
 - ▶ Only approximate coverage is guaranteed
 - ▶ Choice of weights or window size affects reliability

2 High-dimensional / complex models

- Deep learning models or multivariate series:
 - ▶ Large training and calibration costs
 - ▶ Nonconformity score definition becomes nontrivial

3 **Multi-step forecasting challenges**

- Recursive multi-step predictions accumulate errors
- Nonconformity scores for future steps are unknown
- Temporal dependencies inherent in multi-step forecasts

4 **Other considerations**

- Trade-off between coverage and interval width
- Choice of calibration set and score function is crucial

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Problem Setup

- A time series $\{y_t\}_{t \geq 1}$ generated by an unknown DGP
- Exogenous predictors $\mathbf{x}_t = (x_{1,t}, \dots, x_{p,t})'$
- Data point $\{z_t = (\mathbf{x}_t, y_t)\}_{t \geq 1} \subseteq \mathbb{R}^p \times \mathbb{R}$
- Forecasting model \hat{f}_t , generating forecasts $\{\hat{y}_{t+h|t}\}_{h \in [H]}$
- Sequential split:
 - ▶ a *proper training set*: $\mathcal{D}_{\text{tr}} \subset \{1, \dots, t_r\}$
 - ▶ a *calibration set* $\mathcal{D}_{\text{cal}} \subset \{t_r + 1, \dots, t_r + t_c\}$, where $t_c \gg H$
- Nonconformity score:

$$s_{t+h|t} = \mathcal{S}(z_{1:t}, y_{t+h}) := y_{t+h} - \hat{f}_t(z_{1:t}, \mathbf{x}_{t+1:h}) = y_{t+h} - \hat{y}_{t+h|t}.$$

Framework: Online Learning with Sequential Splits

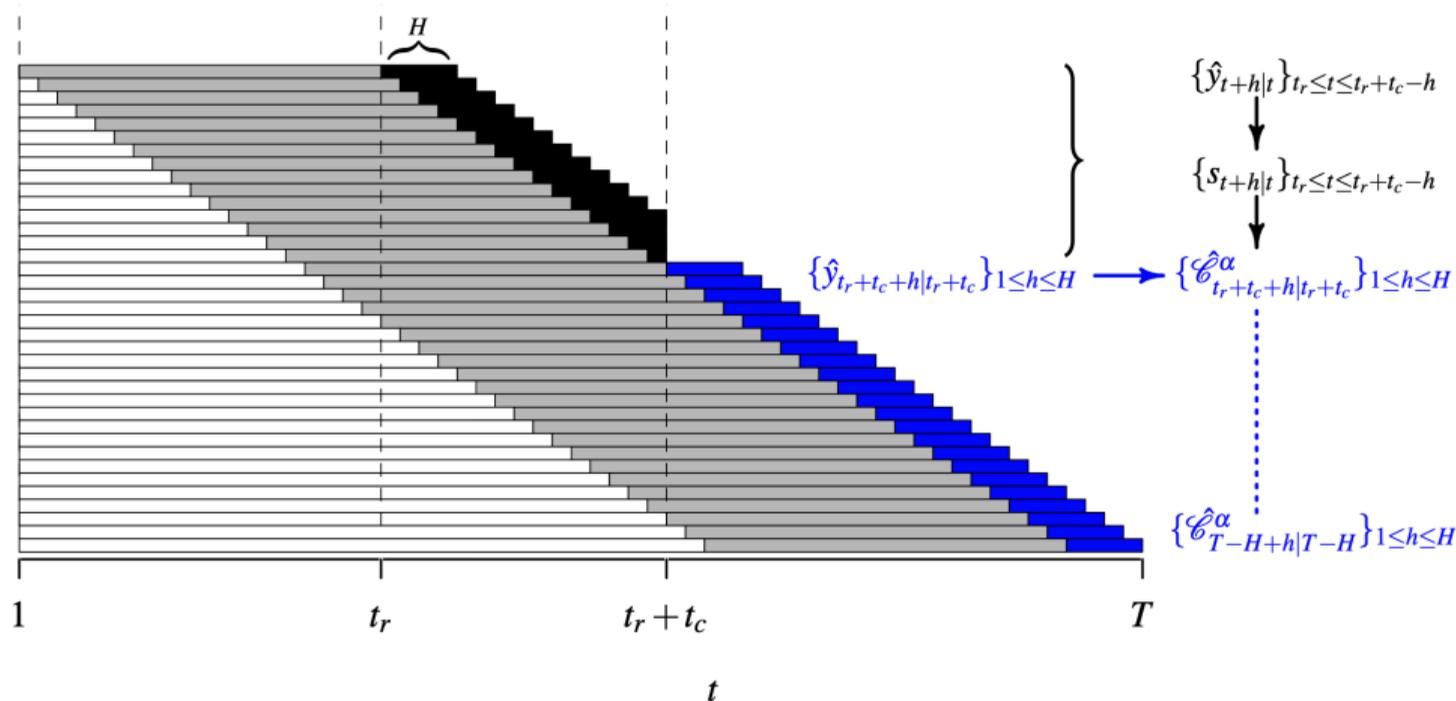


Figure 1: Diagram of the online learning framework with sequential splits. White: unused data; Gray: training data; Black: forecasts in calibration set; Blue: forecasts in test set.

■ Multi-step Split Conformal Prediction (MSCP)

$$\hat{\mathcal{C}}_{t+h|t}^\alpha = \left\{ \mathbf{y} \in \mathbb{R} : \mathbf{s}_{t+h|t}^y \leq Q_{1-\alpha} \left(\sum_{i=t-t_c+1}^t \frac{1}{t_c+1} \cdot \delta_{\mathbf{S}_i|j-h} + \frac{1}{t_c+1} \cdot \delta_{+\infty} \right) \right\}. \quad (1)$$

■ Multi-step Split Conformal Prediction (MSCP)

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■ Multi-step Weighted Conformal Prediction (MWCP)

We choose fixed weights $w_i = b^{t+1-i}$, $b \in (0, 1)$ and $i = t - t_c + 1, \dots, t$.

$$\hat{C}_{t+h|t}^\alpha = \left\{ y \in \mathbb{R} : s_{t+h|t}^y \leq Q_{1-\alpha} \left(\sum_{i=t-t_c+1}^t \tilde{w}_i \cdot \delta_{s_{i|i-h}} + \tilde{w}_{t+1} \cdot \delta_{+\infty} \right) \right\},$$

where \tilde{w}_i and \tilde{w}_{t+1} are normalized weights given by

$$\tilde{w}_i = \frac{w_i}{\sum_{i=t-t_c+1}^t w_i + 1}, \text{ for } i \in \{t-t_c+1, \dots, t\} \quad \text{and} \quad \tilde{w}_{t+1} = \frac{1}{\sum_{i=t-t_c+1}^t w_i + 1}.$$

■ Multi-step Adaptive Conformal Prediction (MACP)

Assuming that:

- ▶ $\beta \mapsto \mathbb{P}(\mathbf{y}_{t+h} \in \hat{\mathcal{C}}_{t+h|t}^\beta)$ is continuous and non-increasing
- ▶ $\mathbb{P}(\mathbf{y}_{t+h} \in \hat{\mathcal{C}}_{t+h|t}^0) = 1$ and $\mathbb{P}(\mathbf{y}_{t+h} \in \hat{\mathcal{C}}_{t+h|t}^1) = 0$
- ▶ an optimal value $\alpha_{t+h|t}^* \in [0, 1]$ exists such that the realised miscoverage rate of the corresponding prediction interval closely approximates the nominal miscoverage rate α .

For each $h \in [H]$, we iteratively estimate $\alpha_{t+h|t}^*$ by updating a parameter $\alpha_{t+h|t}$ through a sequential adjustment process

$$\alpha_{t+h|t} := \alpha_{t+h-1|t-1} + \gamma(\alpha - \text{err}_{t|t-h}),$$

where $\gamma > 0$ is a fixed step size parameter. Then the h -step-ahead prediction interval is computed using Equation (1) by setting $\alpha = \alpha_{t+h|t}$.

■ Multi-step conformal PID control (MPID)

For each $h \in [H]$, the iteration of the h -step-ahead quantile estimate is given by

$$q_{t+h|t} = \underbrace{q_{t+h-1|t-1} + \eta(\text{err}_{t|t-h} - \alpha)}_{\text{P (quantile tracking)}} + \underbrace{r_t \left(\sum_{i=h+1}^t (\text{err}_{i|i-h} - \alpha) \right)}_{\text{I (error integration)}} + \underbrace{\hat{S}_{t+h|t}}_{\text{D (scorecasting)}},$$

where $\eta > 0$ is a constant learning rate, and r_t is a saturation function that adheres to the following conditions

$$x \geq c \cdot g(t-h) \Rightarrow r_t(x) \geq b, \quad \text{and} \quad x \leq -c \cdot g(t-h) \Rightarrow r_t(x) \leq -b, \quad (2)$$

for constant $b, c > 0$, and an admissible function g that is sublinear, nonnegative, and nondecreasing.

Properties of Multi-step Forecast Errors

Assume a time series $\{y_t\}_{t \geq 1}$ generated by a non-stationary AR process:

$$y_t = f_t(y_{(t-d):(t-1)}, \mathbf{x}_{(t-k):t}) + \varepsilon_t, \quad (3)$$

where f_t is a nonlinear function, and ε_t is white noise.

- The sequence of model coefficients that parameterizes the function f is restricted to ensure that the stochastic process is locally stationary.

Based on Wold's representation theorem, for a zero-mean covariance-stationary time series, the optimal linear least-squares forecasts have h -step-ahead errors that are at most MA($h - 1$) process (Diebold, 2024; Harvey et al., 1997).

Proposition 1 (MA($h - 1$) process for h -step-ahead optimal forecast errors)

Let $\{y_t\}_{t \geq 1}$ be a time series generated by a general non-stationary autoregressive process as given in Equation (3), and assume that any exogenous predictors are known into the future. Then the forecast errors of optimal h -step-ahead forecasts follow an approximate MA($h - 1$) process

$$e_{t+h|t} = \omega_{t+h} + \theta_1 \omega_{t+h-1} + \dots + \theta_{h-1} \omega_{t+1}.$$

where ω_t is white noise.

Proposition 2 (Autocorrelations of multi-step optimal forecast errors)

Let $\{y_t\}_{t \geq 1}$ be a time series generated by a general non-stationary autoregressive process as given in Equation (3), and assume that any exogenous predictors are known into the future. The forecast errors for optimal h -step-ahead forecasts can be approximately expressed as

$$e_{t+h|t} = \omega_{t+h} + \phi_1 e_{t+h-1|t} + \dots + \phi_p e_{t+h-p|t},$$

where $p = \min\{d, h - 1\}$, and ω_t is white noise. Therefore, the optimal h -step-ahead forecast errors are at most serially correlated to lag $(h - 1)$.

Autocorrelated Multi-step Conformal Prediction (AcMCP)

For each $h \in [H]$, the iteration of the h -step-ahead quantile estimate is given by

$$q_{t+h|t} = q_{t+h-1|t-1} + \eta(\text{err}_{t|t-h} - \alpha) + r_t \left(\sum_{i=h+1}^t (\text{err}_{i|i-h} - \alpha) \right) + \tilde{e}_{t+h|t}.$$

- $\tilde{e}_{t+h|t}$ is a forecast combination of two simple models:
 - ▶ an MA($h - 1$) model trained on the h -step-ahead forecast errors available up to and including time t (i.e., $e_{1+h|1}, \dots, e_{t|t-h}$)
 - ▶ a linear regression model trained by regressing $e_{t+h|t}$ on forecast errors from past steps (i.e., $e_{t+h-1|t}, \dots, e_{t+1|t}$).

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Proposition 3

Let $\{s_{t+h|t}\}_{t \in \mathbb{N}}$ be any sequence of numbers in $[-b, b]$ for any $h \in [H]$, where $b > 0$, and may be infinite. Assume that r_t is a saturation function obeying Equation (2), for an admissible function g . Then the iteration $q_{t+h|t} = r_t \left(\sum_{i=h+1}^t (\text{err}_{i|i-h} - \alpha) \right)$ satisfies

$$\left| \frac{1}{T-h} \sum_{t=h+1}^T (\text{err}_{t|t-h} - \alpha) \right| \leq \frac{c \cdot g(T-h) + h}{T-h}, \text{ for any } T \geq h+1.$$

Therefore the prediction intervals obtained by the iteration yield the correct long-run coverage; i.e., $\lim_{T \rightarrow \infty} \frac{1}{T-h} \sum_{t=h+1}^T \text{err}_{t|t-h} = \alpha$.

Corollary 1

Let $\{s_{t+h|t}\}_{t \in \mathbb{N}}$ be any sequence of numbers in $[-b, b]$ for any $h \in [H]$, where $b > 0$, and may be infinite. Then the iteration

$q_{t+h|t} = q_{t+h-1|t-1} + \eta(\text{err}_{t|t-h} - \alpha)$ satisfies

$$\left| \frac{1}{T-h} \sum_{t=h+1}^T (\text{err}_{t|t-h} - \alpha) \right| \leq \frac{b + \eta h}{\eta(T-h)}, \text{ for any } \eta > 0 \text{ and } T \geq h + 1.$$

Therefore the prediction intervals obtained by the iteration yield the correct long-run coverage; i.e., $\lim_{T \rightarrow \infty} \frac{1}{T-h} \sum_{t=h+1}^T \text{err}_{t|t-h} = \alpha$.

Corollary 2

Let $\{\hat{q}_{t+h|t}\}_{t \in \mathbb{N}}$ be any sequence of numbers in $[-\frac{b}{2}, \frac{b}{2}]$, and $\{s_{t+h|t}\}_{t \in \mathbb{N}}$ be any sequence of numbers in $[-\frac{b}{2}, \frac{b}{2}]$, for any $h \in [H]$, $b > 0$ and may be infinite. Assume that r_t is a saturation function obeying Equation (2), for an admissible function g .

Then the prediction intervals obtained by the AcMCP iteration yield the correct long-run coverage; i.e., $\lim_{T \rightarrow \infty} \frac{1}{T-h} \sum_{t=h+1}^T \text{err}_{t|t-h} = \alpha$.

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Simulated Linear Autoregressive Process

Consider a simulated stationary ts generated from an AR(2) process:

$$y_t = 0.8y_{t-1} - 0.5y_{t-2} + \varepsilon_t,$$

where ε_t is white noise with error variance $\sigma^2 = 1$.

- $N = 5000$ data points
- \mathcal{D}_{tr} and \mathcal{D}_{cal} , each with a length of 500
- $H = 3$
- Fit AR(2) models

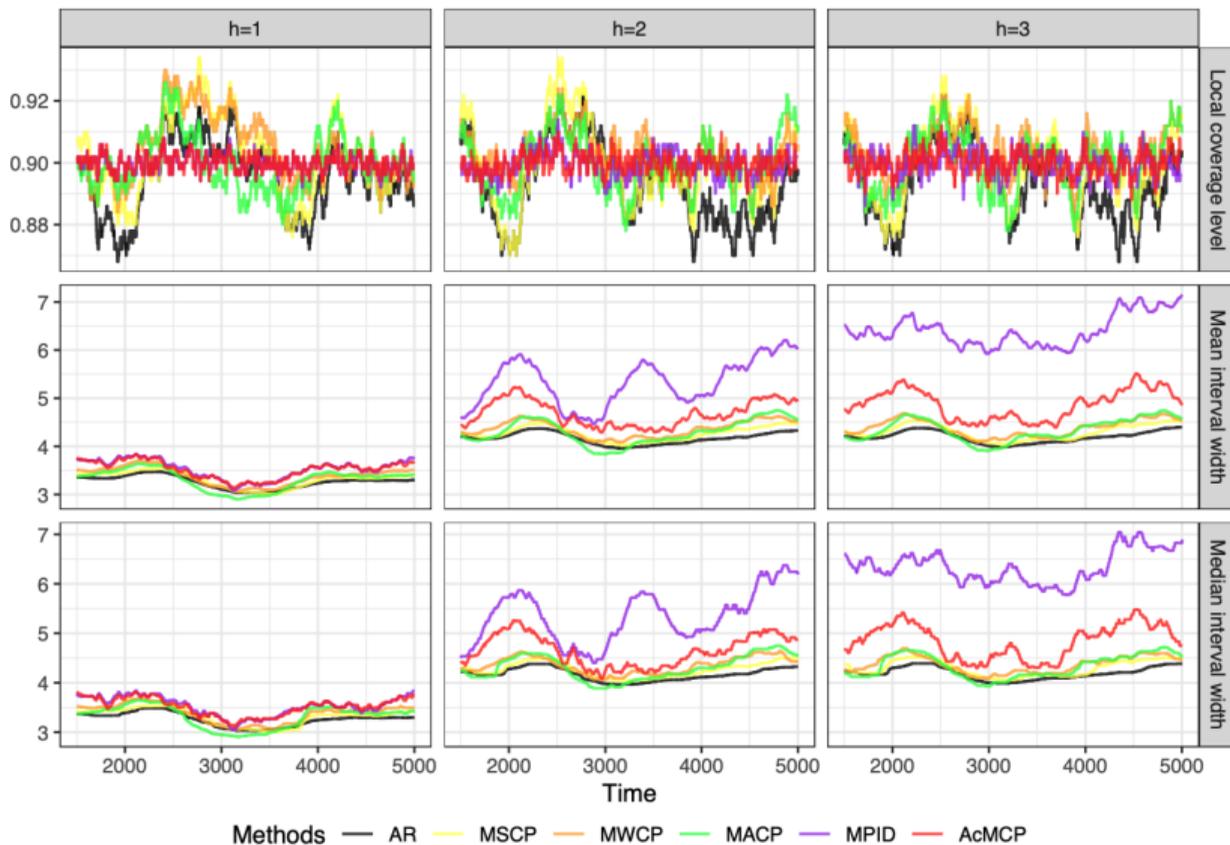


Figure 2: *AR(2) simulation results showing rolling coverage, mean and median interval width for each forecast horizon. The displayed curves are smoothed over a rolling window of size 500. The target coverage level is $1 - \alpha = 0.9$.*

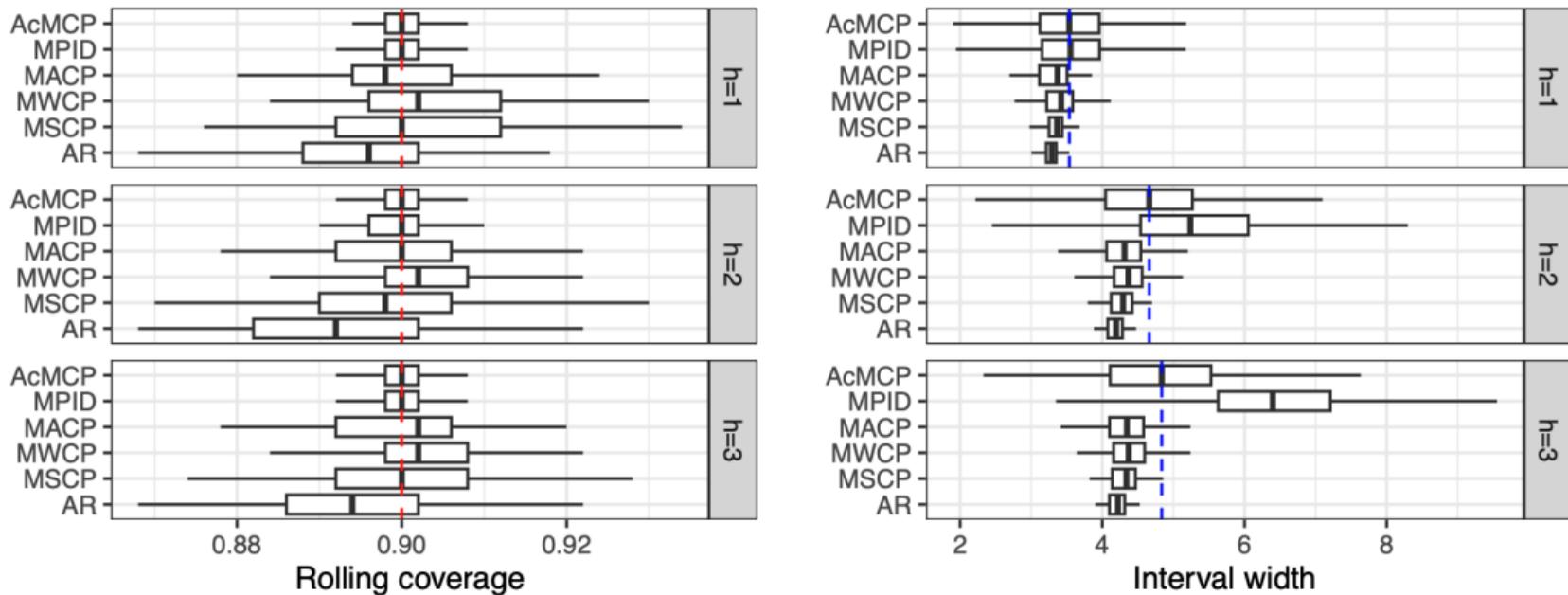


Figure 3: *AR(2) simulation results showing boxplots of the rolling coverage and interval width for each method across different forecast horizons. The red dashed lines show the target coverage level, while the blue dashed lines indicate the median interval width of the AcMCP method.*

Simulated Nonlinear Autoregressive Process

Consider a nonlinear data generation process:

$$y_t = \sin(y_{t-1}) + 0.5 \log(y_{t-2} + 1) + 0.1y_{t-1}x_{1,t} + 0.3x_{2,t} + \varepsilon_t,$$

where $x_{1,t}$ and $x_{2,t}$ are uniformly distributed on $[0, 1]$, and ε_t is white noise with error variance $\sigma^2 = 0.1$.

- $N = 2000$ data points
- \mathcal{D}_{tr} and \mathcal{D}_{cal} , each with a length of 500
- $H = 3$
- Fit feed-forward neural networks with a single hidden layer and lagged inputs

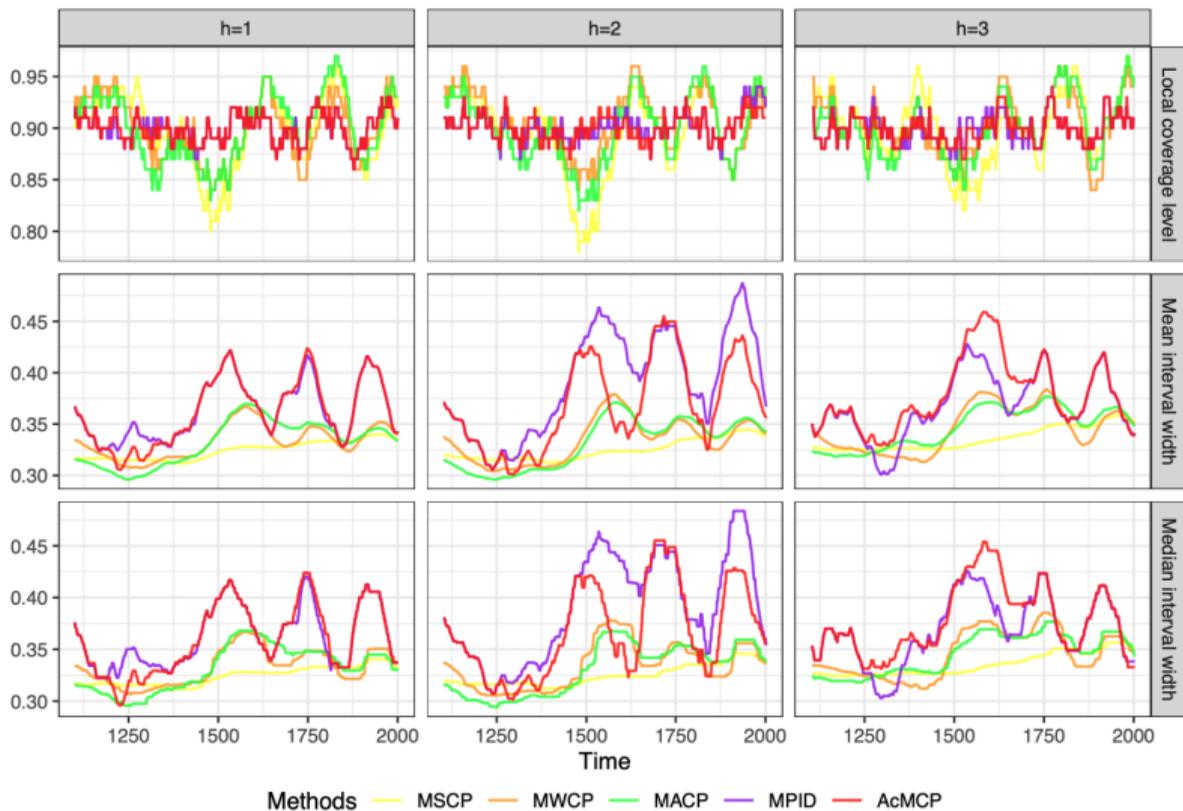


Figure 5: Nonlinear simulation results showing rolling coverage, mean and median interval width for each forecast horizon. The displayed curves are smoothed over a rolling window of size 100. The target coverage level is $1 - \alpha = 0.9$.

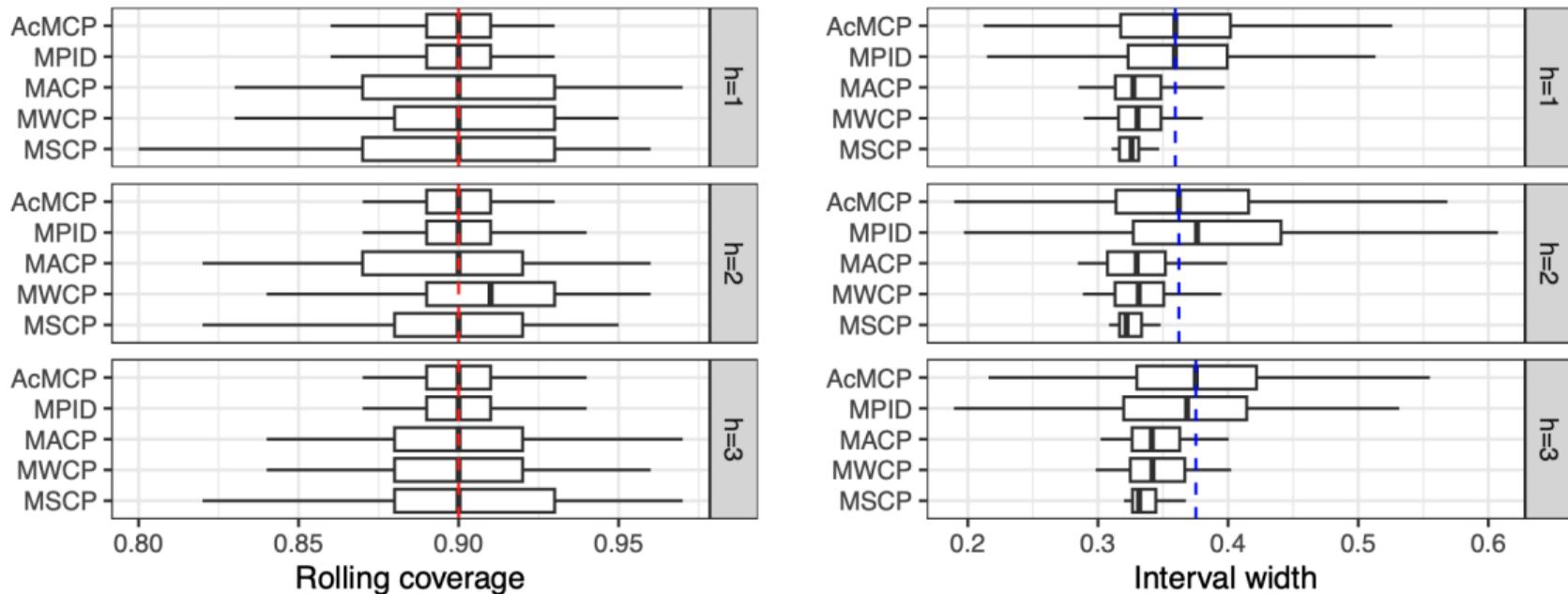


Figure 6: *Nonlinear simulation results showing boxplots of the rolling coverage and interval width for each method across different forecast horizons. The red dashed lines show the target coverage level, while the blue dashed lines indicate the median interval width of the AcMCP method.*

Eating Out Expenditure Data

The data involves **monthly** expenditure on cafes, restaurants and takeaway food services in Victoria from April 1982 up to December 2019.

Forecasting:

- $\mathcal{D}_{\text{tr}} = 20$ years, $\mathcal{D}_{\text{cal}} = 5$ years, and $\mathcal{D}_{\text{test}} = 152$ months
- $H = 12$
- Fit ARIMA with logarithmic transformation, ETS, and STL-ETS, and then output their simple average as final point forecasts

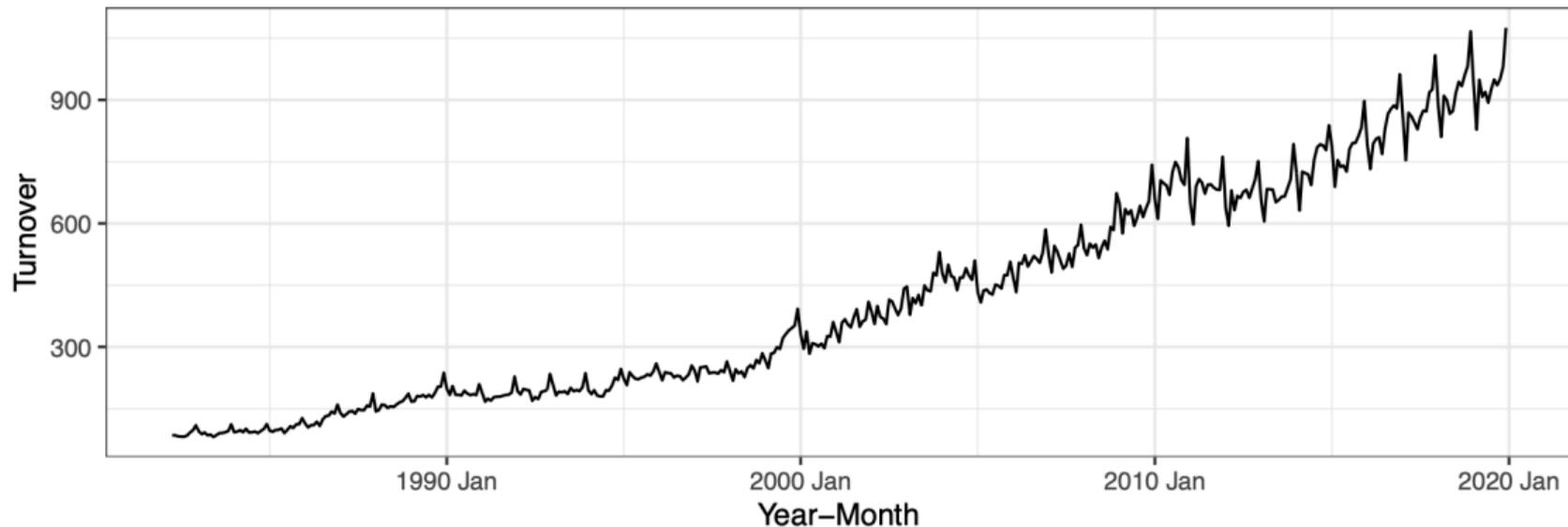


Figure 11: *Monthly expenditure on cafes, restaurants and takeaway food services in Victoria, Australia, from April 1982 to December 2019.*

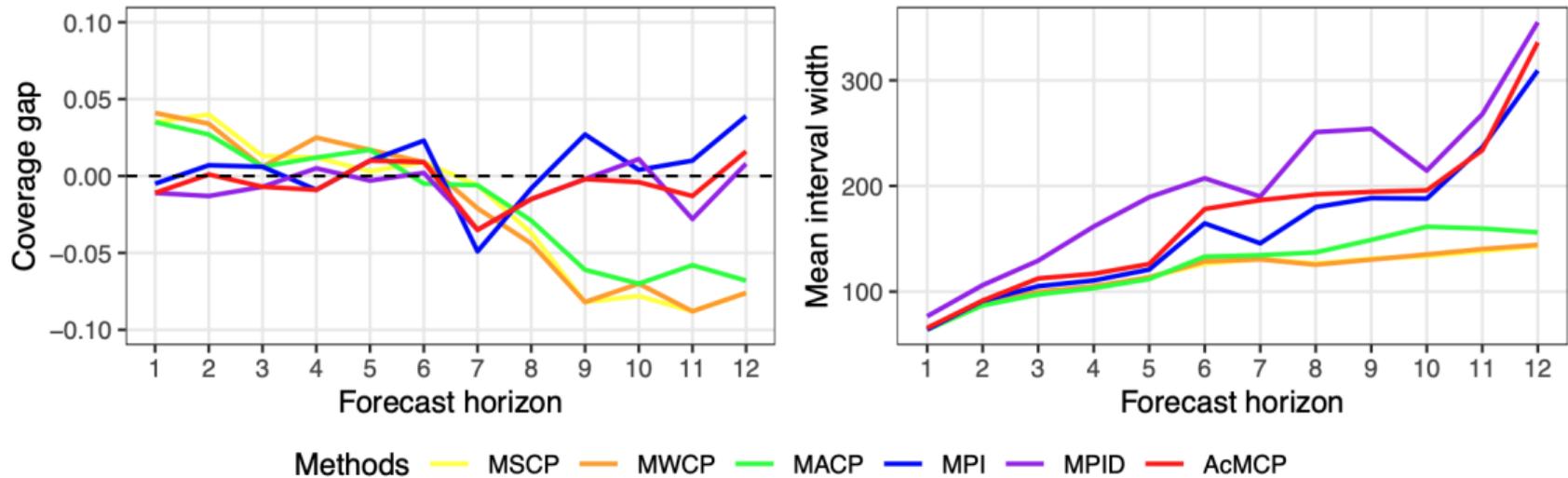


Figure 12: *Eating out expenditure data results showing coverage gap and interval width averaged over the entire test set for each forecast horizon. The black dashed line in the top panel indicates no difference from the 90% target level.*

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Takeaways and Extensions

Takeaways:

- Introduced the core ideas of conformal prediction
- Reviewed several classical conformal prediction methods
- Discussed how these ideas can be extended to time series settings

Further extensions:

- Limited to ex-post forecasting
- Trade-off between coverage and interval width

More Information

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Find me at ...

-  `xqnwang.rbind.io`
-  `@xqnwang`
-  `xiaoqian.wang@amss.ac.cn`

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